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BUOYANCY EFFECTS ON TURBULENT TRANSPORT IN COMBINED FREE AND FORCED CONVECTION BETWEEN VERTICAL PARALLEL PLATES

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Abstract — Turbulent mixed flow of free and forced convection was investigated experimentally and theoretically. Experiments were carried out in the upward turbulent flow between vertical parallel plates at different wall temperatures. In this condition, the aiding and opposing flows arose simultaneously on the heated and cooled sides, respectively, and a fully developed condition was established.

Buoyancy effects on the mean velocity and temperature profiles, eddy diffusivities of heat and momentum, Nusselt number and friction factor, as well as the intensity of velocity and temperature fluctuations were examined, and the substantial effects of the buoyancy force were confirmed.

In addition, an analytical model was proposed, based upon the damping factor concept of the turbulent fluctuations due to viscous action and buoyancy force. It is confirmed that the analytical model predicts the experimental results for the buoyancy effects on the turbulent transport process, and also partly explains the behavior of the turbulent properties.

NOMENCLATURE

A^+, damping constant [–];
b, distance between parallel plates [m];
b^+, dimensionless distance between parallel plates $= bu^*/\nu_c$ [–];
c_p, specific heat at constant pressure [J kg^{-1} K^{-1}];
D, damping factor [–];
F, gain function [–];
f_c, friction factor $= 2|\tau_c|/\langle\rho_c\rangle\langle u\rangle_c^2$ [–];
f_H, friction factor $= 2|\tau_H|/\langle\rho\rangle_H \langle u\rangle_H^2$ [–];
G, buoyancy parameter $= -g\beta e^{i\phi}\, T_0/\nu$ [–];
G^+, buoyancy parameter $= Gr_q/(Pr\, b^{+4})$ [–];
Gr, Grashof number based on temperature difference $= g\langle\beta\rangle\,(T_H - T_c)(2b)^3/\langle\nu\rangle^2$ [–];
Gr_q, Grashof number based on heat flux $= g\beta_c|q|b^4/(\nu_c^2\lambda_c)$ [–];
g, gravitational acceleration [m s^{-2}];
i, imaginary number $= \sqrt{-1}$;
k, von Karman constant [–];
l, mixing length [m];
Nu_c, Nusselt number $= 4|q|\delta/(\langle T\rangle_c - T_c)/\langle\lambda\rangle_c$ [–];
Nu_H, Nusselt number $= 4q|(b-\delta)/(T_H - \langle T\rangle_H)/\langle\lambda\rangle_H$ [–];
$\mathscr{P}$, static pressure [Pa];
p^+, dimensionless pressure $= (\mathscr{P} + \rho_c g x)/\rho_c u^{*2}$ [–];
Pr, Prandtl number $= c_p\mu_c/\lambda_c$ [–];
q, heat flux [J/m^2 s];
Re, Reynolds number $= 2\langle u\rangle b/\langle\nu\rangle$ [–];
T, temperature [K];
T^+, dimensionless temperature normalized by the cooled-wall parameter $= (T - T_c)\rho_c c_p u^*/|q|$ [–];
T_H^+, dimensionless temperature normalized by the heated-wall parameter $= (T_H - T)\rho_H c_p u_H^*/|q|$ [–];
T_c^*, $|q|/\rho_c c_p u_c^*$ [K];
T_H^*, $|q|/\rho_H c_p u_H^*$ [K];
t, time [s];
u, time averaged velocity in x direction [m s^{-1}];
u^+, dimensionless velocity $= u/u^*$ [–];
u^*, friction velocity $= \sqrt{|\tau_c|/\rho_c}$ [m s^{-1}];
v, velocity in y direction [m s^{-1}];
x, coordinate in the flow direction [m];
x^+, dimensionless coordinate in the flow direction $= xu^*/\nu_c$;
y, distance from the cooled wall [m];
y^+, dimensionless distance from the cooled wall $= yu^*/\nu_c$ [–];
z, correction factor defined by equation (19) [–].

Greek symbols

α, heat flux parameter $= |q|/T_c\rho_c c_p u_c^*$ [–];

β, thermal expansion coefficient [K^{-1}];
δ, distance from the cooled wall to the maximum-velocity location [m];
δ^+, dimensionless distance $= \delta u^*/\nu_c$ [−];
ε_m, eddy diffusivity of momentum [$m^2 s^{-1}$];
ε_h, eddy diffusivity of heat [$m^2 s^{-1}$];
λ, thermal conductivity [$J m^{-1} K^{-1} s^{-1}$];
μ, viscosity [$kg m^{-1} s^{-1}$];
ν, kinematic viscosity [$m^2 s^{-1}$];
ρ, density [$kg m^{-3}$];
τ, shear stress [Pa];
τ^+, dimensionless shear stress $=\tau/|\tau_c|$ [−];
ϕ, phase shift between velocity and temperature fluctuations [−];
ω, angular velocity [s^{-1}].

Subscripts and superscripts
C, at the cooled wall;
H, at the heated wall;
h, heat;
m, momentum;
0, forced convection;
$\langle\ \rangle$, averaged over the cross section, $0 \leqq y \leqq b$;
$\langle\ \rangle_C$, averaged over the cooled side, $0 \leqq y \leqq \delta$;
$\langle\ \rangle_H$, averaged over the heated side, $\delta \leqq y \leqq b$;
$+$, normalized by wall parameters;
$'$, fluctuating component.

1. INTRODUCTION

THE PRESENCE of a temperature difference in a forced-flow field gives rise to density differences and thus to a buoyancy force. The buoyancy force may be expected to influence the turbulent transport phenomena of heat and momentum when the buoyancy force is considerably greater than those accompanying the forced flow. Depending upon whether the buoyancy force is acting to aid or oppose the forced flow, the flow is referred to as aiding or opposing flow. For such combined free and forced convection, that is, mixed convection, Brown and Gauvin [1, 2] investigated the influence of buoyancy force upon the heat transfer rates and temperature fluctuations in aiding and opposing flows by using a vertical heated tube. Several other investigations [3–6] have been made concerning mixed convection in a vertical circular tube or over a vertical plane surface. Those results are, in general, that the opposing flow promotes the turbulent heat transfer rates, while the aiding flow inhibits them. In those flow configurations, however, it is difficult to clarify the effects of the buoyancy force upon the turbulent transport phenomena, since the flows are developing and not in a fully developed condition.

Among the analytical approaches to mixed convection which have been proposed previously, the following two are prominent. One assumes that the expressions for the distribution of eddy diffusivities in the forced convection can be applied to mixed convection by taking into account the temperature dependence of the fluid properties. But Khosla *et al.* [7] showed that the eddy diffusivity models for the forced convection were found to be inadequate and the buoyancy force has substantial influence on the turbulent transport process. Oosthuizen [8] presented another method in which the effects of the buoyancy force on the turbulent transport can be predicted by extending the mixing length model. However, it contains a constant for evaluating the buoyancy effects, which should be determined by experiment. In addition, although the mixing length model is useful in fully turbulent flow, mixed convection generally occurs at low Reynolds numbers and so the buoyancy effects should be considered together with viscous effects. The present study attempts to predict buoyancy effects on the turbulent transport process in mixed convection, in both aiding and opposing flows, by extending the damping factor model given by van Driest [9]. Experiments were carried out on the mixed convection between vertical parallel plates at different temperatures, for which the flow was fully developed. This experimental data was compared with the predicted results. In addition, the intensities of temperature and velocity fluctuations were observed in order to supplement the discussion of the turbulent mixed convection.

2. ANALYSIS

2.1. *Governing equations*

Basic equations for the fully-developed combined flow between vertical parallel plates maintained at different temperatures, as shown in Fig. 1, are given as follows. The fluid is flowing upwards. If the Boussinesq approximation is made, the equations which govern this motion can be written as

$$\frac{d\tau}{dy} = -\frac{d}{dx}(\mathscr{P} + \rho_C g x) + \rho_C \beta_C g (T - T_C), \quad (1)$$

$$\tau = -(\mu + \rho\varepsilon_m)\frac{du}{dy}, \quad (2)$$

where the density ρ is approximated by the first two terms of the Taylor series which is expanded in temperature T around the reference temperature T_C at the cooled wall. Because the flow is fully developed and the two walls are maintained at constant temperatures, the heat flux is uniform over the cross-section. Hence, the basic equation of heat transfer is

$$q = -(\lambda + \rho c_p \varepsilon_h)\frac{dT}{dy} = \text{constant}. \quad (3)$$

Boundary conditions are given as follows,

$$\left.\begin{array}{lll} u = 0 & \text{and } T = T_C & \text{at } y = 0 \\ u = 0 & \text{and } T = T_H & \text{at } y = b \\ q = \text{constant} & & \text{for all } y. \end{array}\right\} \quad (4)$$

Eddy diffusivities, ε_m and ε_h, for momentum and heat transfer, respectively appearing in equations (2) and (3), are to be determined by the theoretical model which will be presented in the next section.

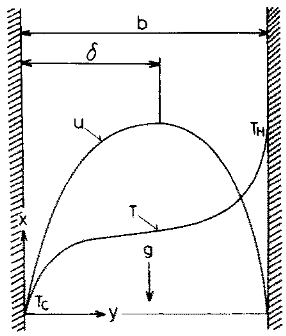

FIG. 1. Schematic diagram of flow and heat transfer.

By introducing the wall parameters at the cooled wall, equations (1), (2) and (3) are normalized as

$$\frac{d\tau^+}{dy^+} = -\frac{dp^+}{dx^+} + G^+T^+, \quad (5)$$

$$\tau^+ = -\left(\frac{\mu}{\mu_C} + \frac{\rho\varepsilon_m}{\rho_C \nu_C}\right)\frac{du^+}{dy^+}, \quad (6)$$

$$l = \left(\frac{1}{Pr}\frac{\lambda}{\lambda_C} + \frac{\rho}{\rho_C}\frac{\varepsilon_h}{\nu_C}\right)\frac{dT^+}{dy^+}, \quad (7)$$

where $G^+ = Gr_q/(Pr \cdot b^{+4})$ and Gr_q is based on the heat flux. Boundary conditions are

$$\left.\begin{array}{ll} u^+ = 0, \quad \tau^+ = -1 \quad \text{and } T^+ = 0 & \text{at } y^+ = 0, \\ u^+ = 0 & \text{at } y^+ = b^+. \end{array}\right\} \quad (8)$$

When examining the buoyancy effects upon transport phenomena, we have to consider the variation of physical properties due to the presence of temperature distribution. Here the temperature dependence of physical properties is approximated by equation (9) for air; as was done in Mizushina *et al.* [10],

$$\left.\begin{array}{l} \dfrac{\rho}{\rho_C} = \dfrac{T_C}{T} = \dfrac{1}{1+\alpha T^+}, \\ \dfrac{\mu}{\mu_C} = \dfrac{\lambda}{\lambda_C} = (1+\alpha T^+)^{0.68}, \end{array}\right\} \quad (9)$$

where the specific heat c_p is assumed to be constant and where $\alpha = |q|/T_C\rho_C c_p u_C^*$.

Here, it is noted that the concept of eddy diffusivities may become inadequate when the velocity or temperature gradient is zero, especially when the Reynolds stress is not equal to zero in spite of the zero velocity gradient. These phenomena are expected to appear in the present flow configuration, since the distributions of mean velocity and turbulent properties are not symmetric at the plane of zero velocity gradient. These facts, however, do not cause a serious error in the predicted mean velocity and temperature profiles and so in the friction factor and the heat transfer rate. Thus, it is assumed here that the eddy diffusivity has a zero value once the velocity gradient vanishes.

Reynolds number, Grashof number, Nusselt number and friction factor are defined by equations (10)–(15). Here, δ is the distance from the cooled wall to the location at which the velocity has its maximum value. Nusselt number and friction factor are defined for both sides of the wall, i.e. the cooled and heated sides, which are bounded by the plane of $y = \delta$. Hence, these dimensionless groups are expressed as

$$Re = 2(1+\alpha\langle T^+\rangle)^{-1.68}\int_0^{b+} u^+\,dy^+, \quad (10)$$

$$Gr = 8\Delta T^+ G^+ b^{+3}(1+\alpha\langle T^+\rangle)^{-3.36}, \quad (11)$$

$$Nu_C = 4\delta^+ Pr/\{\langle T^+\rangle_C(1+\alpha\langle T^+\rangle_C)^{0.68}\}, \quad (12)$$

$$Nu_H = 4(b^+ - \delta^+)Pr/\{(\Delta T^+ - \langle T^+\rangle_H)(1+\alpha\langle T^+\rangle_H)^{0.68}\}, \quad (13)$$

$$f_C = 2(1+\alpha\langle T^+\rangle_C)\left\{\delta^+\Big/\int_0^{\delta+} u^+\,dy^+\right\}^2, \quad (14)$$

$$f_H = 2(1+\alpha\Delta T^+)^{2.36}(1+\alpha\langle T^+\rangle_H) \times\left\{z(b^+ - \delta^+)\Big/\int_{\delta^+}^{b+} u^+\,dy^+\right\}^2, \quad (15)$$

where $\Delta T = T_H - T_C$ and

$$\langle T^+\rangle = \int_0^{b+} T^+u^+\,dy^+\Big/\int_0^{b+} u^+\,dy^+, \quad (16)$$

$$\langle T^+\rangle_C = \int_0^{\delta+} T^+u^+\,dy^+\Big/\int_0^{\delta+} u^+\,dy^+, \quad (17)$$

$$\langle T^+\rangle_H = \int_{\delta^+}^{b+} T^+u^+\,dy^+\Big/\int_{\delta^+}^{b+} u^+\,dy^+, \quad (18)$$

$$z = \frac{\nu_C}{\nu_H}\frac{u}{u} = \frac{\nu_C}{\nu_H}\sqrt{\left(\frac{|\tau_H|}{|\tau_C|}\frac{\rho_C}{\rho_H}\right)}. \quad (19)$$

Here again it should be noted that the normalization was made with respect to the cooled-wall parameters.

2.2. *Damping factor model*

Van Driest [9] successfully derived a model to account for the damping of turbulence caused by molecular viscosity action near a solid surface. He assumed that the viscous damping of turbulent fluid oscillation near the solid surface was represented by $(1 - F)$, where F was the gain function of the motion while an infinite flat plate was undergoing simple harmonic oscillation parallel to the plate in an infinite fluid.

Following the same line of physical reasoning, here, we extend his model to the non-isothermal field where the buoyancy force as well as the viscous force are predominant. Basic equations are written as

$$\frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial y^2} \pm \beta g T, \quad (20)$$

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c_p}\frac{\partial^2 T}{\partial y^2}. \quad (21)$$

The positive buoyancy term corresponds to the aiding flow. The boundary conditions are

$$\left.\begin{array}{lll} u = U_0 e^{i\omega t} & \text{and } T = T_0 e^{i(\omega t+\phi)} & \text{at } y = 0, \\ u = T = 0 & & \text{at } y = \infty. \end{array}\right\} \quad (22)$$

By assuming that there is no temperature dependence of physical properties, the solutions of equation (20) and (21) subject to these boundary conditions are given by the following expressions:

$$u = U_0 e^{i\omega t}\left(\exp\left[-\sqrt{\left(\frac{i\omega}{\nu}\right)}y\right] \pm \frac{\nu G}{i\omega U_0(1-Pr)}\right.$$
$$\left.\times\left\{\exp\left[-\sqrt{\left(\frac{i\omega}{\nu}\right)}y\right] - \exp\left[-\sqrt{\left(\frac{i\omega}{\nu}Pr\right)}y\right]\right\}\right), \quad (23)$$

$$T = T_0 e^{i(\omega t+\phi)}\exp\left[-\sqrt{\left(\frac{i\omega}{\nu}Pr\right)}y\right], \quad (24)$$

where $G = -g\beta e^{i\phi} T_0/\nu$. Then the gain function F of equation (23) is

$$F = \left|\exp\left[-\sqrt{\left(\frac{i\omega}{\nu}\right)}y\right] \pm \frac{\nu G}{i\omega U_0(1-Pr)}\right.$$
$$\left.\times\left\{\exp\left[-\sqrt{\left(\frac{i\omega}{\nu}\right)}y\right] - \exp\left[-\sqrt{\left(\frac{i\omega}{\nu}Pr\right)}y\right]\right\}\right|.$$

Hence, from a similar treatment as van Driest's, the damping factor of momentum transfer for the mixed convection is expressed as

$$D_m = 1 - \left|\exp(-Ey^+/A^+) \pm \frac{\nu G}{i\omega U_0(1-Pr)}\right.$$
$$\left.\times\{\exp(-Ey^+/A^+) - \exp[-E\sqrt{(Pr)}y^+/A^+]\}\right|, \quad (25)$$

where $A^+ = u^*\sqrt{(2/\omega\nu)}$ and $E = 1 + i$. In addition, the buoyancy term $\nu G/\omega U_0$ is rewritten as

$$\frac{\nu G}{\omega U_0} = -\tfrac{1}{2}A^{+2}G^+\frac{e^{i\omega}T_0\rho c_p u^*/|q|}{U_0/u^*}. \quad (26)$$

The damping factor of heat transfer is not influenced by the buoyancy and is given by

$$D_h = 1 - \exp[-\sqrt{(Pr)}y^+/A^+]. \quad (27)$$

Tanimoto and Hanratty [11] discussed the relation between the distributions of temperature and velocity fluctuations in the wall region, and showed that those distributions were similar, both in their magnitude and in their variation with distance from the wall. As $e^{i\phi}T_0$ and U_0 correspond to the intensities of temperature and velocity fluctuations, respectively, the following approximate relation in the wall region is assumed;

$$\frac{e^{i\phi}T_0\rho c_p u^*/|q|}{U_0/u^*} = Pr. \quad (28)$$

Substitution of equations (26) and (28) into equation (25) gives

$$D_m = 1 - \left|\exp(-Ey^+/A^+) \pm \frac{i}{2}A^{+2}G^+\frac{Pr}{1-Pr}\right.$$
$$\left.\times\{\exp(-Ey^+/A^+) - \exp[-E\sqrt{(Pr)}y^+/A^+]\}\right|. \quad (29)$$

For the mixing length the following expressions similar to those proposed by Cebeci [12] were devised for the forced convection at low Reynolds number,

$$l_m = k_m y \quad \text{and} \quad l_h = k_h y \quad (30)$$

where

$$k_m = 0.4\left(0.974 - \frac{20.77}{\delta^+ - 5.48}\right), \quad (31)$$

$$k_h = 0.44\left(1.026 + \frac{14.4}{\delta^+ + 10.0}\right). \quad (32)$$

When $\delta < y < b$, i.e. in the heated side, y and δ^+ are replaced by $b - y$ and $(b^+ - \delta^+)z\ (= b_H^+ - \delta_H^+)$, where z is the correction factor to normalize the variables by means of the wall parameters of the cooled side. In addition, the damping constant A^+ is given by

$$A^+ = 26\left(1.026 + \frac{20.77}{\delta^+ - 5.48}\right). \quad (33)$$

Hence, the eddy diffusivities for the forced convection are

$$\frac{\varepsilon_m}{\nu_C} = (k_m y^+ D_{m0})^2\left|\frac{du^+}{dy^+}\right|, \quad (34)$$

$$\frac{\varepsilon_h}{\nu_C} = k_m k_h (y^+)^2 D_{m0} D_{h0}\left|\frac{du^+}{dy^+}\right|, \quad (35)$$

where $D_{m0} = 1 - \exp(-y^+/A^+)$ and $D_{h0} = 1 - \exp[-\sqrt{(Pr)}y^+/A^+]$. By the application of equations (30)–(35) to forced convection, the predicted distributions of velocity, temperature and eddy diffusivities for heat and momentum transfer were in good agreement with the experimental results.

For combined free and forced convection, the eddy diffusivities are expressed by equation (29) as

$$\frac{\varepsilon_m}{\nu_C} = (k_m y^+ D_m)^2\left|\frac{du^+}{dy^+}\right|, \quad (36)$$

$$\frac{\varepsilon_h}{\nu_C} = k_m k_h (y^+ D_m)^2\frac{D_{h0}}{D_{m0}}\left|\frac{du^+}{dy^+}\right|. \quad (37)$$

As will be seen later, the distribution of the turbulent Prandtl number is not influenced very much by the buoyancy force so that here the relation for the turbulent Prandtl number in nearly-isothermal flow is assumed applicable to the mixed convection.

3. EXPERIMENTAL APPARATUS AND PROCEDURE

The experimental apparatus is shown schematically in Fig. 2. Measurements were carried out in a 10.5 : 1 rectangular duct made of stainless steel with a cross-section of 0.21 × 0.02 m. The duct was 4.5 m in length. The test section was located 4 m downstream from the entrance and the inlet length was about 100 times the

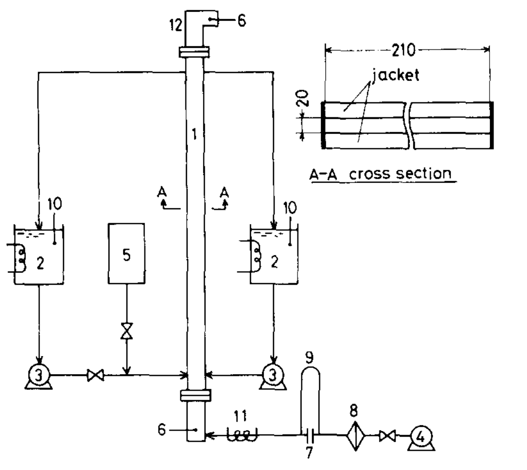

FIG. 2. Experimental apparatus; 1. Test section. 2. Reservoir. 3. Pump. 4. Blower. 5. Electric boiler. 6. Thermocouple. 7. Orifice meter. 8. Air filter. 9. Manometer. 10. Regulator. 11. Preheater. 12. Mixing box.

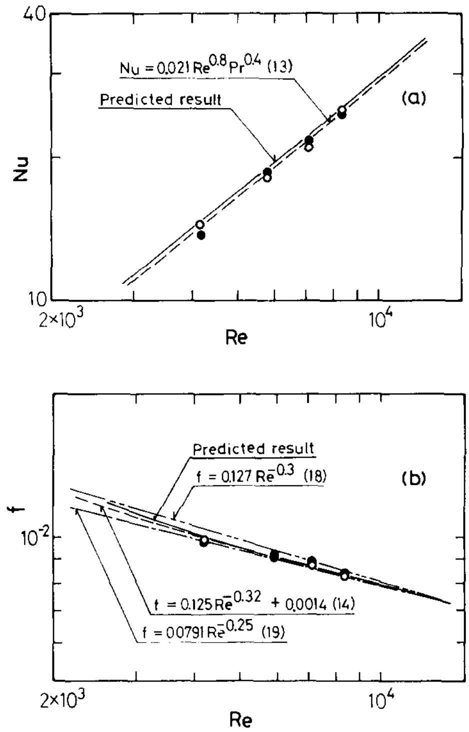

FIG. 3. Nusselt number and friction factor for forced convection.

equivalent diameter. There were two jackets behind the heat transfer surface, as shown in Fig. 2. Through one jacket, cooled water was circulated to maintain the cooled wall at constant temperature. Steam or heated water flowed through the other jacket to maintain the wall at higher temperature. Side walls were made of phenolic-resin plates 1 mm thick in order to prevent heat conduction through the walls. Air was introduced upwards into the duct and discharged from the duct into the atmosphere. The difference of the bulk temperature between inlet and outlet sections was adjusted to be as small as possible in order to establish the fully-developed temperature distribution in a shorter entrance length.

The measurements of velocity and temperature, both mean and fluctuating quantities, were made with wire anemometer probes and circuits (Hayakawa HC-310). The main part of the anemometer probe consisted to two tungsten wires with diameter 5 μm, one of which was 1 mm in length and used as a hot wire. The other one was 1.2 mm in length and used as a cold wire with constant current (1 mA). The cold wire was located 0.7 mm upstream from the hot wire, and the velocity and temperature could be measured simultaneously in the presence of high temperature variance and fluctuation.

The experiments were made at $Re = 4170$ to 8270, and $Gr/Re^2 = 1.12 \times 10^{-3}$ to 1.82×10^{-2}. Heat flux and shear stress at the wall were measured from the gradients of temperature and velocity near the wall, respectively.

4. RESULTS AND DISCUSSION

4.1. *Turbulent transport in the mixed convection*

Measurements of Nusselt number and friction factor were shown in Figs. 3(a) and (b) for nearly-isothermal forced convection in which the difference between the cooled and the heated wall temperature was less than 10°C. Solid and open circles represented the data in the heated and cooled sides, respectively, but coincided with each other. Solid lines in each figure were values predicted by using the mixing length and damping factor for forced convection, i.e. equations (30)–(35). The experimental results were in good agreement with the predicted curves and the familiar correlations, that is, the formula of McAdams [13] for the heat transfer and that of Drew *et al.* [14] for the friction factor:

$$Nu_0 = 0.021\, Re^{0.8}\, Pr^{0.4}, \tag{38}$$

$$f_0 = 0.125\, Re^{-0.32} + 0.0014. \tag{39}$$

Thus, equations (30)–(35) were reasonable for adoption to turbulent transport at low Reynolds number. A more crucial comparison will be given later for the eddy diffusivities and confirm the validity of the prediction.

For mixed flow of free and forced convection, experimental results of Nusselt number and friction factor are shown in Figs. 4(a) and (b). By considering the set of normalized equations (5)–(7) together with the damping factor model of turbulence, equation (29), the single parameter to represent the buoyancy effects is seen to be G^+, that is, Gr/Re^2. Thus, the results were plotted in the form of Nu/Nu_0 and f/f_0 against Gr/Re^2. Experimental results agree acceptably well with the predicted values. On the cooled side, the reduction in friction factor and increase in Nusselt

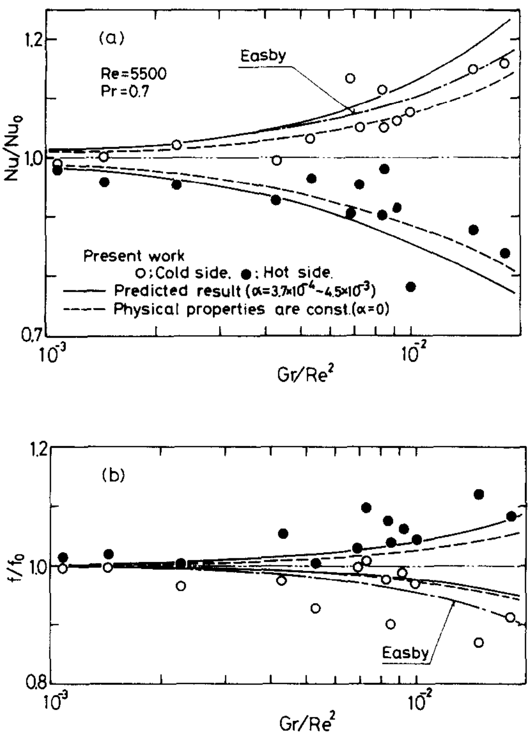

FIG. 4. Nusselt number and friction factor for mixed flow.

number can be attributed to the promotion of turbulent transfer of heat and momentum as the buoyancy force increases, because the buoyancy force is exerted in a direction reverse to the flow and the unstable convection is caused in the field. On the heated side, the buoyancy causes an increase in the friction factor and a decrease in the Nusselt number. Easby [15] measured the friction factor and Nusselt number for downward flow of Nitrogen gas in a vertical heated pipe and proposed the empirical correlations; $f/f_0 = 1.006 - 5.13\,Gr/Re^2$ and $Nu/Nu_0 = 1.009 + 8.91$

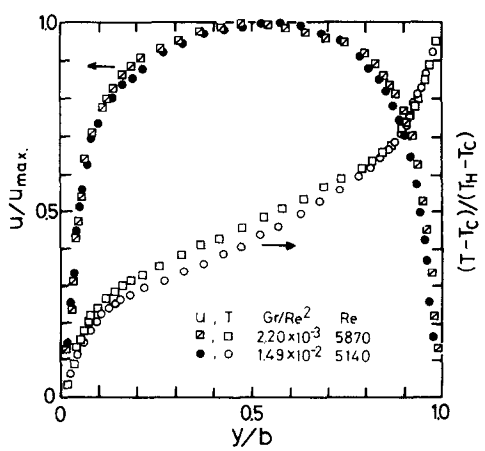

FIG. 5. Mean velocity and temperature profiles.

Gr/Re^2. This flow configuration corresponds to that on the cooled side of the present experiment, and these results agree quantitatively with each other.

Typical distributions of the mean velocity and temperature for mixed convection are shown in Fig. 5. The gradient of the mean velocity on the heated side is very much steeper than on the cooled side, and so the profile is skewed asymmetrically. This profile is similar in form to those found in annular flow and duct flow for walls of different roughness. A similar tendency is also seen in the gradient of mean temperature.

The predicted and experimental distributions of temperature and velocity for mixed convection were normalized by the respective wall parameters and are shown in Figs. 6 (a) and (b). As the temperature difference increases, the distributions of dimensionless temperature become flatter at $y^+ > 20$ on the cooled side and steeper on the heated side than for the forced convection. However, in contrast with Fig. 5, the velocity profiles normalized by the wall parameters are not influenced much by the buoyancy force. The possible reason for this is that in spite of the remarkable change in eddy diffusivity caused by the buoyancy force, the force balance tends to deform the shear stress distribution and results in a suppression of the buoyancy effect on the normalized velocity distribution.

The measured and predicted eddy diffusivities for heat and momentum transfer were plotted in Figs. 7 (a) and (b), in which the solid curves were calculated by the present damping-factor model. Eddy diffusivities increase on the cooled side and decrease on the heated side as the Gr/Re^2 increases. The predicted curves represented fairly well such variations in eddy diffusivities.

4.2. *Turbulence quantities in the mixed convection*

Measured intensities of the axial velocity fluctuations when the temperature difference between the cooled and heated walls is less than 10°C are presented in Figs. 8 and 9. They are in good agreement, both in their magnitude and their variation with distance from the wall, with the results obtained by Kreplin [16] in oil flow between parallel plates.

For mixed convection the distribution of the axial velocity intensity changes with Gr/Re^2 and are compared with the nearly-isothermal results in Figs. 10 and 11. Here again the normalization for the heated and cooled sides was done with the respective wall parameters. The buoyancy force works, in general, to reduce the turbulence intensities in the aiding flow, while in the opposing flow case, it causes the enhancement of the turbulence intensity. Carr *et al.* [17] measured the turbulence intensity in the aiding flow, that is, in the up-flow of air in a vertical heated pipe with constant heat flux at $Re \simeq 5000$. They indicated that the axial velocity intensity was reduced across the pipe cross-section as the heat flux increased and so the peak value near the wall was reduced. Similar phenomena are seen for the heated side of Fig. 11. On the

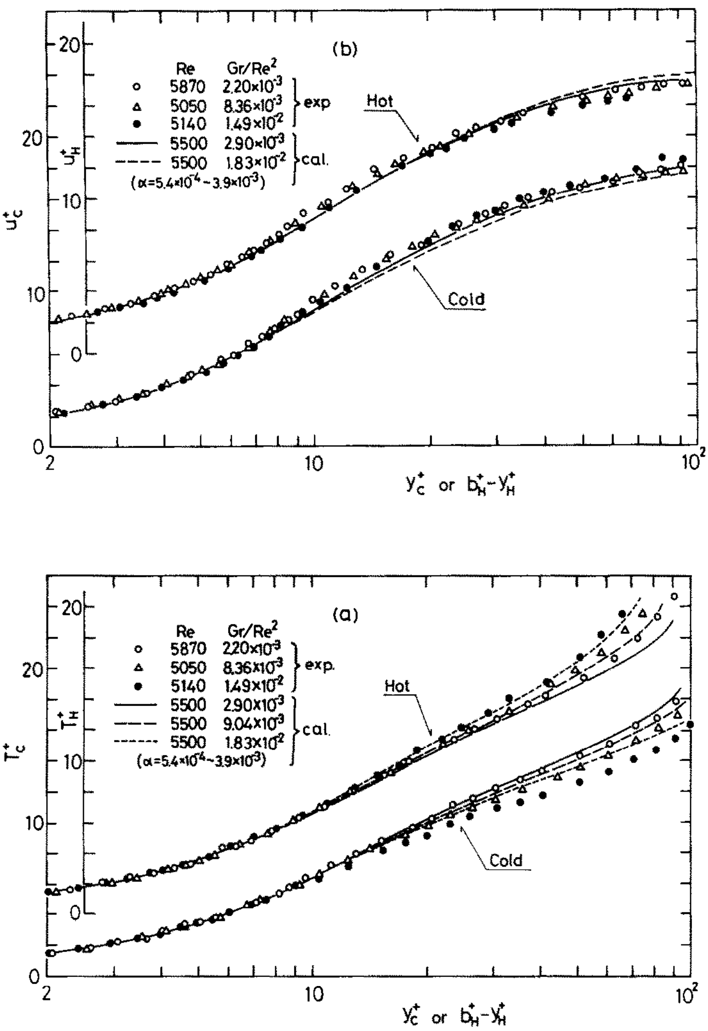

FIG. 6. Mean velocity and temperature profiles normalized with wall parameters.

cooled side the turbulence intensity increases remarkably near the wall.

Distributions of the temperature fluctuation intensity are shown in Figs. 12 and 13. In contrast to the turbulence intensity, it is not possible to recognize the effects of buoyancy in the wall region when the temperature fluctuation intensities are normalized by the wall parameters. This is expected from the fact that the damping factor for temperature fluctuation, i.e., equation (27), is not influenced by buoyancy. However, in the fully turbulent region, $0.2 < y/\delta < 0.8$ in Fig. 13, remarkable buoyancy effects on the temperature fluctuation intensity exist. This is strongly related with the heat transfer configuration investigated here, that is, heat flows uniformly from the heated wall to the cooled wall.

In the fully turbulent region the transport equation of the temperature fluctuation intensity may be written as

$$\frac{D\overline{T'^2}}{Dt} = -\overline{v'T'}\frac{\partial T}{\partial y} - 2\frac{\lambda}{\rho c_p}\overline{\frac{\partial T'}{\partial x_k}\frac{\partial T'}{\partial x_k}} - \frac{\partial}{\partial x_k}\left(\overline{u_k'T'^2} - \frac{\lambda}{\rho c_p}\frac{\partial \overline{T'^2}}{\partial x_k}\right), \quad (40)$$

where the summation convention is used with respect to repeated indices. In the above equation the convective term vanishes in this case because the flow is fully developed. The first term of the right side represents the production rate of $\overline{T'^2}$, the second is the dissipation rate and the last term denotes the diffusive transport. As seen in Fig. 13 the diffusive transport may be neglected because of the uniform distribution of $\overline{T'^2}$, and so the production rate balances with the dissipation rate.

Since the heat flux is uniform across the flow field, i.e. $-\rho c_p \overline{v'T'}$ is constant, the production of $\overline{T'^2}$ is

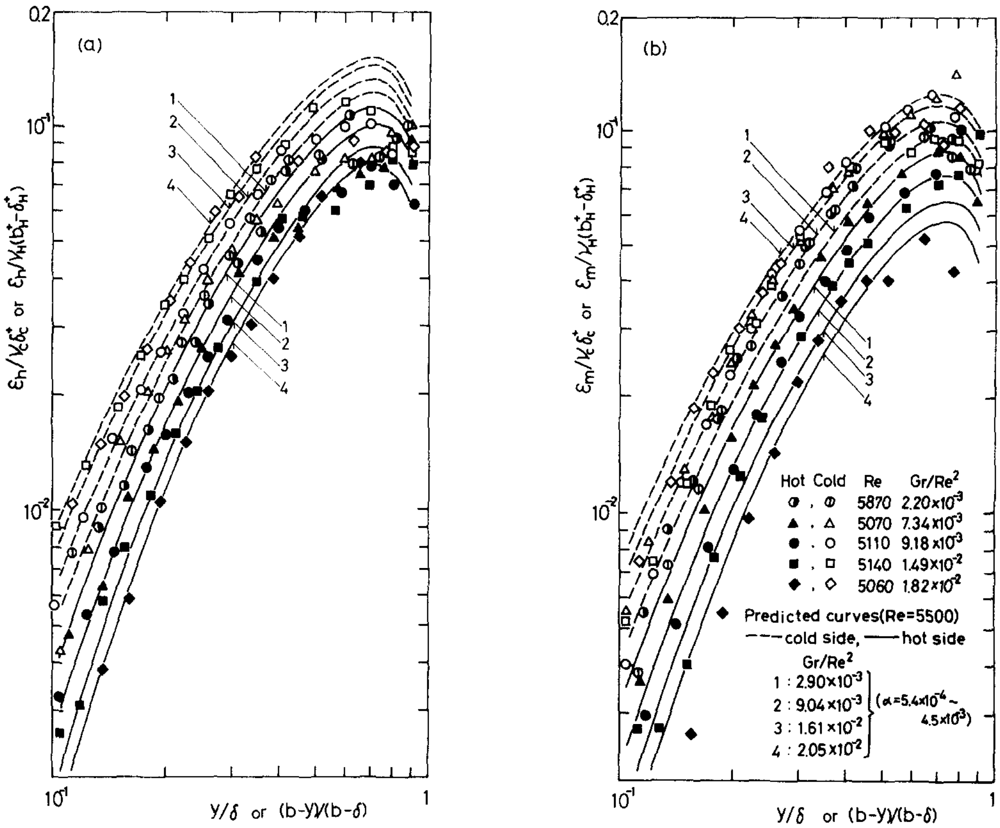

FIG. 7. Eddy diffusivities of heat and momentum for mixed flow.

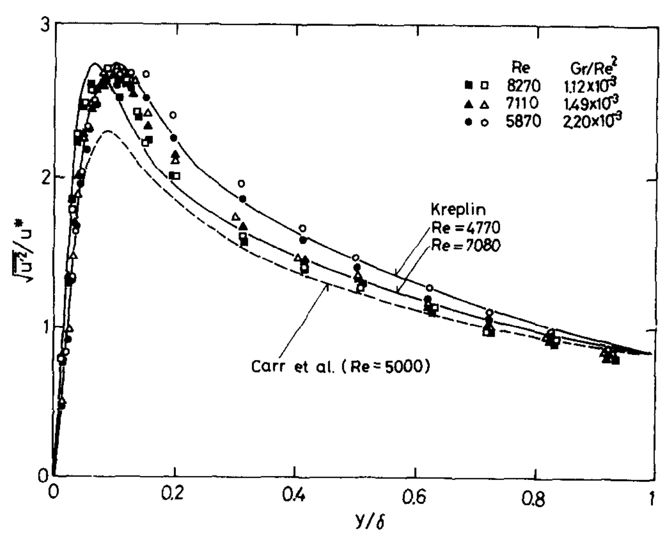

FIG. 8. Intensity of the axial velocity fluctuation for forced convection.

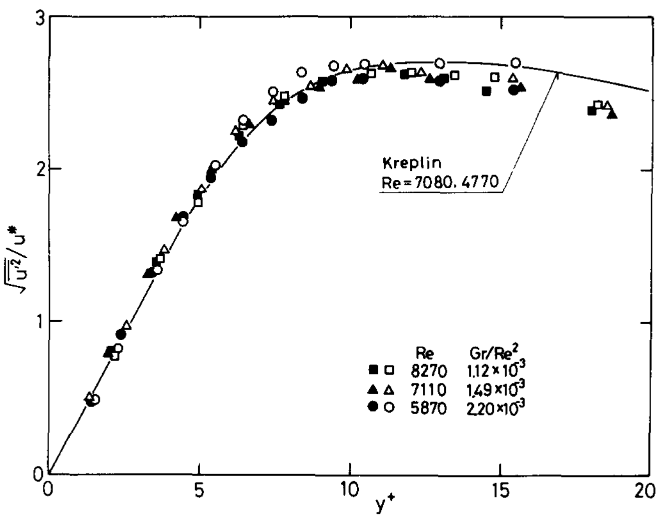

FIG. 9. Intensity of the axial velocity fluctuation in the wall region of forced convection.

proportional to $\partial T/\partial y$. On the cooled side, that is, in the opposing flow, the eddy diffusivities increase and so the mean temperature gradient is reduced. This causes a lower production rate of $\overline{T'^2}$ and is believed to result in the smaller level of the temperature fluctuation intensity. From this reasoning, $\overline{T'^2}$ decreases in the opposing flow and increases in the aiding flow.

CONCLUSION

To study the turbulent mixed flow of free and forced convection, experiments were carried out for vertical parallel plates whose wall temperatures are held constant but at different values. The fully-developed condition was attained in this flow configuration. Substantial effects of the buoyancy force arising from the temperature distribution are confirmed for the turbulent transport processes of heat and momentum, and it was found that it deforms the distributions of mean velocity and temperature asymmetrically.

In the aiding flow, in which the buoyancy force acts in the flow direction, eddy diffusivities of heat and momentum are reduced remarkably. In the opposing flow, they increase with increasing Gr/Re^2, which is derived from the dimensionless equation of motion. In spite of this, when normalized by the wall parameters, the velocity profiles are not influenced as much because of the total balance among the shear stress, the

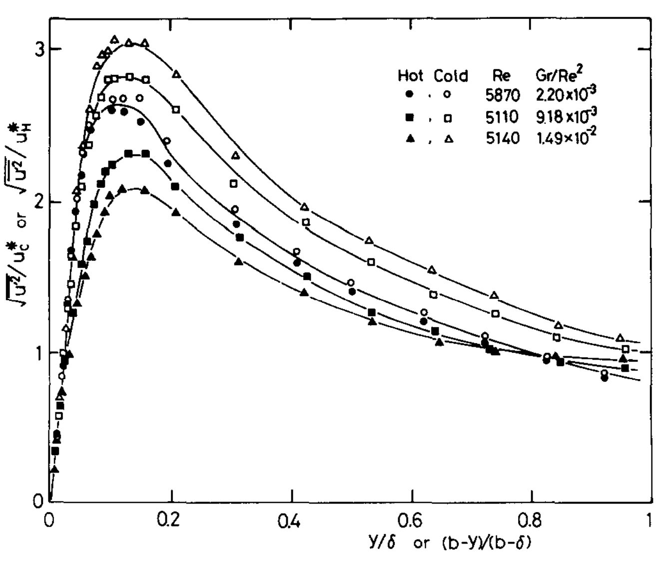

FIG. 10. Intensity of the axial velocity fluctuation for mixed flow.

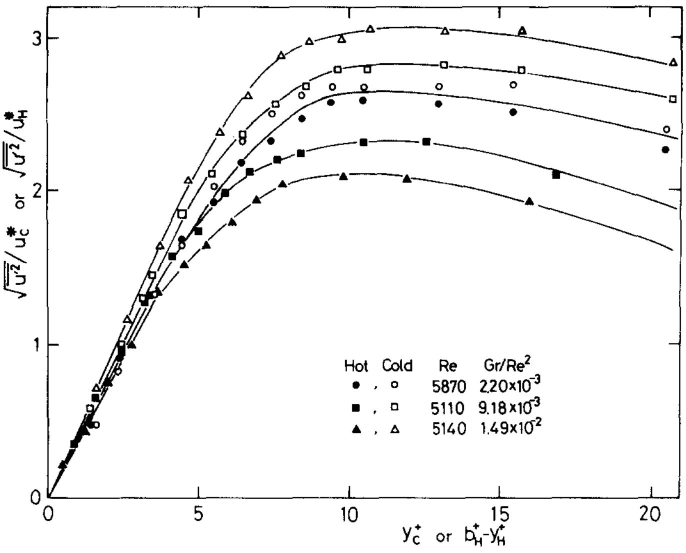

FIG. 11. Intensity of the axial velocity fluctuation in the wall region of mixed flow.

fluid pressure and the buoyancy force. The Nusselt number decreases and the friction factor increases in the aiding flow, and vice versa in the opposing flow.

An analytical model was proposed, based upon the damping factor concept of the turbulent fluctuations due to the viscous action and the buoyancy force. It predicts the experimental results very well, and also explains the behavior of the turbulence properties; the intensity of the axial velocity fluctuation increases in the opposing flow and decreases in the aiding flow. However, the intensity of the temperature fluctuation in the wall region is not influenced seriously by the buoyancy force.

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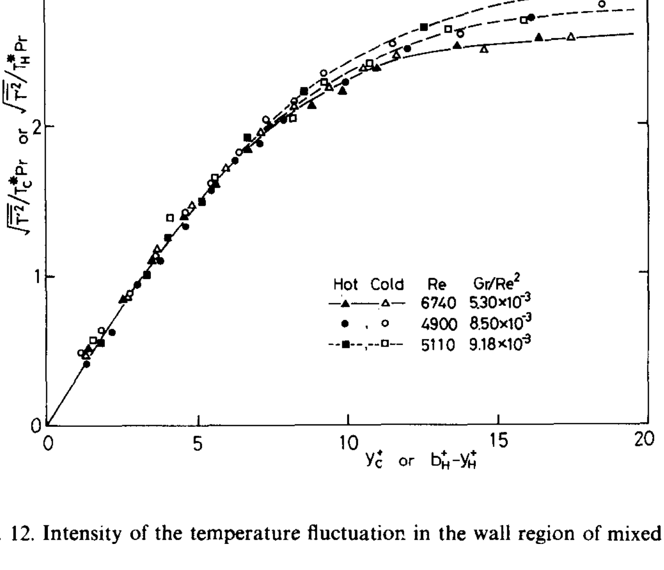

FIG. 12. Intensity of the temperature fluctuation in the wall region of mixed flow.

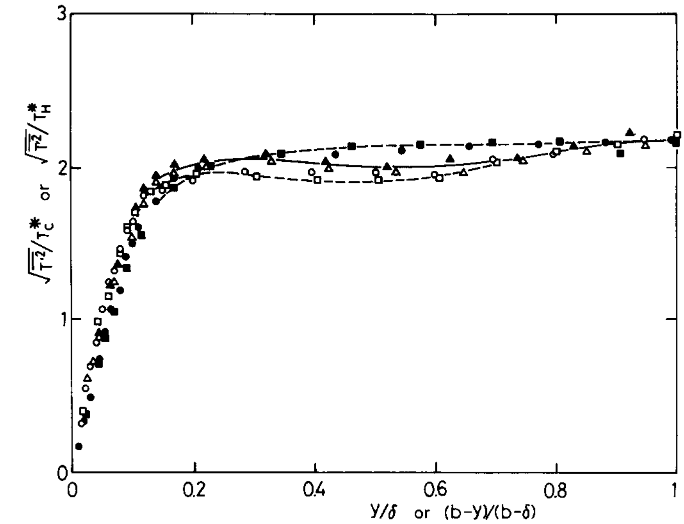

FIG. 13. Intensity of the temperature fluctuation for mixed flow. Symbols same as in Fig. 12.

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EFFET DES FORCES ARCHIMEDIENNES SUR LE TRANSPORT TURBULENT EN CONVECTION MIXTE ENTRE DEUX PLAQUES VERTICALES ET PARALLELES

Résumé—On étudie expérimentalement et théoriquement la convection mixte turbulente. Les expériences concernent l'écoulement turbulent ascendant entre deux plans verticaux parallèles à des températures différentes. Dans ces conditions, un écoulement aide sur la surface chaude, un autre contrarie à la surface froide et une condition complétement établi est réalisée.

Les effets d'Archimède sur les profiles de vitesse et de température moyennes, les diffusivités turbulentes de chaleur et de quantité de mouvement, le nombre de Nusselt et le facteur de frottement sont examinés ainsi que l'intensité des fluctuations de vitesse et de température, et on constate qu'ils sont sensibles.

Un modèle analytique est proposé, basé sur le concept de facteur d'amortissement des fluctuations turbulentes du aux actions de la viscosité et des forces d'Archimède. Le modèle prédit les résultats expérimentaux en ce qui concerne le mécanisme du transport turbulent et il explique en partie le comportement des propriétés turbulentes.

AUFTRIEBSWIRKUNGEN TURBULENTER STRÖMUNGSBEWEGUNG IN ZUSAMMENGESETZTER FREIER UND ERZWUNGENER KONVEKTION ZWISCHEN SENKRECHTEN PARALLELEN PLATTEN

Zusammenfassung—Turbulent gemischte Strömung mit freier und erzwungener Konvektion wurde versuchsmäßig und theoretisch untersucht. Zu der turbulenten Aufwärtsströmung zwischen senkrechten parallelen Platten wurden bei verschiedenen Wandtemperaturen Versuche durchgeführt. Bei diesen Bedingungen stellten sich die sich unterstützenden und entgegenwirkenden Ströme gleichzeitig an der beheizten beziehungsweise gekühlten. Seite ein. Es wurden sowohl die Auftriebswirkungen auf die mittleren Geschwindigkeits- und Temperaturprofile, die turbulenten Transportgrößen für Wärme- und Impulsstrom, die Nusselt-Zahlen und Reibungsbeiwerte als auch auf die Stärke der Geschwindigkeits- und Temperaturschwankungen untersucht und die wesentlichen Einflüsse der Auftriebswirkungen festgestellt. Zusätzlich wurde ein analytisches Modell vorgeschlagen, das auf dem Konzept des Dämpfungsfaktors der turbulenten Schwankung beruht, welcher von Zähigkeitseinflüssen und Auftriebskräften abhängt. Es wird gezeigt, daß das analytische Modell die Versuchsergebnisse für die Auftriebswirkungen auf den turbulenten Transportvorgang gut wiedergibt und ebenso das Verhalten der turbulenten Transportgrößen teilweise erklärt.

ВЛИЯНИЕ АРХИМЕДОВЫХ СИЛ НА ТУРБУЛЕНТНЫЙ ПЕРЕНОС ПРИ СОВМЕСТНОМ ДЕЙСТВИИ СВОБОДНОЙ И ВЫНУЖДЕННОЙ КОНВЕКЦИИ МЕЖДУ ВЕРТИКАЛЬНЫМИ ПАРАЛЛЕЛЬНЫМИ ПЛАСТИНАМИ

Аннотация — Экспериментально и теоретически исследовалось турбулентное смешанное течение с учетом свободной и вынужденной конвекции. Эксперименты проводились в направленном вверх турбулентном потоке между вертикальными параллельными пластинами, находящимися при различной температуре. При этом на нагреваемой стенке возникал спутный поток, а на охлаждаемой — противоток, и создавались условия для полностью развитого режима.

Исследовалось влияние архимедовых сил на профили средней скорости и температуры, вихревую вязкость, вихревую температуропроводность, число Нуссельта и коэффициент трения, а также пульсационные характеристики тепло- и массообмена, причем отмечена существенная роль архимедовых сил.

Кроме того, предложена аналитическая модель, учитывающая затухание турбулентных пульсаций под действием вязкостных и подъемных сил. Показано, что аналитическая модель подтверждает экспериментальные результаты по влиянию архимедовой силы на турбулентный перенос, а также позволяет частично объяснить изменение турбулентных характеристик.